Week 10 (solutions)

Wednesday, March 31, 2021 9:02 AM

4.9 Problems

Problem 1. *Use Composite Simpson's rule and the given value of n to approximate the following improper integrals:*

1. $\int_0^1 x^{-1/4}\sin(x)dx,\ n=4$
2. $\int_0^1 \frac{e^{2x}}{x^{2/5}}dx,\ n=6$

Problem 2. *Use the transformation $t=x^{-1}$ and the composite Simpson's rule for $n=4$ to compute:*

$$\int_1^{\infty}\frac{1}{x^2+9}dx$$

Problem 1.
part.1

Fourth order polynomial for $\sin(x)$ is $x-\frac{x^3}{3!}:=P_4(x)$
write integral as:

$$\int_0^1 \frac{\sin(x)}{x^{\frac{1}{4}}}dx=\int_0^1 \frac{\sin(x)-P_4(x)}{x^{\frac{1}{4}}}dx+\int_0^1 \frac{\mathrm{P}_4(\mathrm{x})}{x^{\frac{1}{4}}}dx$$

second integral is:

$$\int_0^1 x^{\frac{3}{4}}-\frac{x^{\frac{11}{4}}}{6}dx=\frac{166}{315}$$

first integral we evaluate using Simpsons' rule:

let $G(x)=\frac{\sin(x)-P_4(x)}{x^{\frac{1}{4}}}$ when $x\neq 0$ and 0 otherwise.

$G(.25)=0.000005745893976556199$
$G(.5)=0.00021768448428408974$
$G(.75)=0.0018158520590864106$
$G(1)=0.008137651474563162$

Therefore

$$\int_0^1 G(x)\,dx\approx\frac{h}{3}\big(G(0)+4G(.25)+2G(.5)+4G(.75)+G(1)\big)=0.0013216176879486026$$

Therefore final appromimation is: .528306

problem 1
part 2.

$2x$ 16
24

fourth order polynomial of $e^{2x} = 1 + 2x + \frac{4x^2}{2} + \frac{8x^3}{6} + \frac{16x^4}{24}$

$$\int_0^1 \frac{P_4(x)}{x^{\frac{2}{5}}} dx = \frac{135665}{32292} \approx 4.20119534249969$$

Let $G(x) = \frac{\left(e^{2x} - P_4(x)\right)}{x^{\frac{2}{5}}}$

$$\begin{Bmatrix} 0 \\ 0.00007431699515210394, \\ 0.0019118480508397207, \\ 0.013127118018662341, \\ 0.052566852736805385, \\ 0.15685989719537793, \\ 0.38905609893065085 \end{Bmatrix}$$

Simpson's rule gives:
0.06545882385237276

final answer:
4.266654166352063

Problem 2

let $t = \frac{1}{x}$

$$dx = -\frac{1}{t^2} dt$$

$$\int_1^0 \frac{-1}{t^{(-2)} + 9} \frac{1}{t^2} dt = \int_0^1 \frac{1}{1 + 9t^2} dt \approx 0.4112648691514671$$

Problem 3. *Use Theorem 5.4 to show that the following initial-value problems have a unique solution, and find the solution:*

1. $y' = y\cos(t)$, $0 \leq t \leq 1$, $y(0) = 1$
2. $y' = -\frac{2}{t}y + t^2 e^t$, $1 \leq t \leq 2$, $y(1) = \sqrt{2}e$

Problem 4. *Show that the given equation implicitly defines a solution. Approximate $y(2)$ using Newton's method:*

$$y' = -\frac{y^3 + y}{(3y^2 + 1)t}$$

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for $1 \le t \le 2$, $y(1) = 1$. For the equation: $y^3y + yt = 2$

Problem 3.
Part 1.

$f(t, y) = y\cos(t)$
for y_1, y_2

$$\frac{|f(t, y_1) - f(t, y_2)|}{|y_1 - y_2|} \le |\partial_y f| = |\cos(t)| \le 1$$

therefore f is Lipschitz, with Lipschitz constant 1, therefore there is a unique solution

(note it suffices to just establish an upperbound on the first derivative)

$y(t) = e^{\sin(t)}$

Problem 3.
part2.

$$f(t, y) = -\frac{2y}{t} + t^2e^t$$

$$\partial_y f = -\frac{1}{t}$$
$1 \le t \le 2$
$$1 \ge \frac{1}{t} \ge \frac{1}{2}$$
therefore $\left|\frac{1}{t}\right| \le 1$
therefore again f is Lipschitz, with Lipschitz constant 1.

$$y(t) = t^{-2}\left(\sqrt{2}e - 9e + e^t(24 + t(-25 + t(12 + (t - 4)t)\right)$$

Problem 4.

$3y^2y't + y^3 + y't + y = 0$
solve for y'

$$y' = \frac{-y^3 - y}{3y^2 t + t}$$

Newton's method: we know that $y'(1) = -\frac{1}{2}$ and $y(1) = 1$

t	y	y'
1	1	-0.5
1.1	0.95	-0.44317
1.2	0.905683	-0.39697
1.3	0.865986	-0.3587
1.4	0.830116	-0.32652
1.5	0.797464	-0.2991
1.6	0.767554	-0.27547
1.7	0.740006	-0.25491
1.8	0.714516	-0.23685
1.9	0.690831	-0.22088
2	0.668743	-0.20665